Confidence interval

According to the board of directors of a club, the income of active members is around \$1200 per month. From the 2500 active members of the club, a sample of 80 was considered, and it was observed that the average income was \$1120 with a standard deviation of \$130. Construct a 95% confidence interval and use it to draw a conclusion about the belief of the board of directors.

Since the sample size is $n = 80$ (large enough), we can use the normal distribution:

$$SE = \frac{s}{\sqrt{n}} = \frac{\$130}{\sqrt{80}} = \frac{\$130}{8.9443} \approx \$14.530$$

For a confidence level of 95%, the critical value is:

$$Z_{\alpha/2} = Z_{0.025} = 1.96$$

$$ME = Z_{\alpha/2} \times SE = 1.96 \times \$14.530 \approx \$28.479$$

Lower limit:

$$\bar{x} - ME = \$1120 - \$28.479 \approx \$1091.521$$

Upper limit:

$$\bar{x} + ME = \$1120 + \$28.479 \approx \$1148.479$$

$$\boxed{(\$1091.52,\ \$1148.48)}$$

The value of $1200 is not included within the calculated confidence interval ($1091.50 to $1148.50). This indicates that, with 95% confidence, the true mean monthly income of active members lies between $1091.50 and $1148.50. Therefore, it is unlikely that the average income is $1200.

The sum of all probabilities is $S = 0.97$. We calculate the missing value:

$$p = 1 - S = 1 - 0.97 = 0.03$$

We update the complete table:

$Y \setminus X$	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$Y = 0$	0.10	0.10	0.04	0.02
$Y = 1$	0.10	0.15	0.05	0.03
$Y = 2$	0.09	0.13	0.02	0.01
$Y = 3$	0.01	0.10	0.02	**0.03**

The total number of goals is $T = X + Y$.
The expectation of T is:

$$E[T] = E[X] + E[Y]$$

Marginal probabilities of X:

$$\begin{aligned}
P(X = 0) &= 0.10 + 0.10 + 0.09 + 0.01 = 0.30 \\
P(X = 1) &= 0.10 + 0.15 + 0.13 + 0.10 = 0.48 \\
P(X = 2) &= 0.04 + 0.05 + 0.02 + 0.02 = 0.13 \\
P(X = 3) &= 0.02 + 0.03 + 0.01 + 0.03 = 0.09
\end{aligned}$$

Marginal probabilities of Y:

$$\begin{aligned}
P(Y = 0) &= 0.10 + 0.10 + 0.04 + 0.02 = 0.26 \\
P(Y = 1) &= 0.10 + 0.15 + 0.05 + 0.03 = 0.33 \\
P(Y = 2) &= 0.09 + 0.13 + 0.02 + 0.01 = 0.25 \\
P(Y = 3) &= 0.01 + 0.10 + 0.02 + 0.03 = 0.16
\end{aligned}$$

Calculation of $E[X]$:

$$\begin{aligned}
E[X] &= \sum_{x=0}^{3} x \cdot P(X = x) \\
&= 0 \cdot 0.30 + 1 \cdot 0.48 + 2 \cdot 0.13 + 3 \cdot 0.09 \\
&= 0 + 0.48 + 0.26 + 0.27 \\
&= 1.01
\end{aligned}$$

Calculation of $E[Y]$:

$$\begin{aligned}
E[Y] &= \sum_{y=0}^{3} y \cdot P(Y = y) \\
&= 0 \cdot 0.26 + 1 \cdot 0.33 + 2 \cdot 0.25 + 3 \cdot 0.16 \\
&= 0 + 0.33 + 0.50 + 0.48 \\
&= 1.31
\end{aligned}$$

Calculate $E[T]$:

$$E[T] = E[X] + E[Y] = 1.01 + 1.31 = 2.32$$

The team X wins when $X > Y$.
Identify the combinations where $X > Y$:

X	Y	$P(X,Y)$
1	0	0.10
2	0	0.04
2	1	0.05
3	0	0.02
3	1	0.03
3	2	0.01

Sum the probabilities of these combinations:

$$\begin{aligned} P(X > Y) &= P(1,0) + P(2,0) + P(2,1) + P(3,0) + P(3,1) + P(3,2) \\ &= 0.10 + 0.04 + 0.05 + 0.02 + 0.03 + 0.01 \\ &= 0.25 \end{aligned}$$

The expected total number of goals is:

$$E[T] = 2.32$$

On average, 2.32 goals are expected in the match.
The probability that team X wins is:

$$P(X > Y) = 0.25$$